ANALYSIS OF INERTIAL PROPERTIES OF THERMAL RECEIVERS WITH VARIABLE THERMOPHYSICAL CHARACTERISTICS

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The dynamical characteristics of thermal receivers with temperature-dependent thermophysical coefficients are investigated.

The dynamical characteristics of thermal receivers analyzed in many papers [1-5] have been obtained on the basis of solving a linear thermal-conductivity problem [6]. The results of these investigations are applicable either for a narrow range of measurements or for the case when the admissible temperature-measurement error permits considering the thermophysical coefficients of the thermal-receiver material as constants.

However, in the general case when a thermal receiver should be used in a broad temperature range and should assure high accuracy in the measurements, it is impossible to neglect the temperature dependence of the thermophysical coefficients of the thermal-receiver material.

To set up a dependence between the measured temperature of the medium and the temperature of the thermal receiver, it is necessary to find the solution of the nonlinear thermal-conductivity equation for boundary conditions of the third kind. For thermal receivers modelled in the form of a body of the simplest geometric shape (infinite plate and cylinder, sphere), this equation is [7]

$$c\,[\theta(r,\ \tau)]\ \gamma\,[\theta(r,\ \tau)]\ \frac{\partial\theta(r,\ \tau)}{\partial\tau} = \frac{1}{r^{2p+1}}\ \frac{\partial}{\partial r}\left\{\lambda\,[\theta(r,\ \tau)]\ r^{2p+1}\ \frac{\partial\theta(r,\ \tau)}{\partial r}\right\} \tag{1}$$

$$(\tau \geqslant 0,\quad 0 \leqslant r \leqslant R).$$

In connection with the impossibility of obtaining a solution in general form when solving the nonlinear thermal-conductivity problem, constraints dictated by specific conditions are ordinarily imposed in the problem, after which the approximate solution is sought.

Because of their small diameter of thickness, the assumption is made in analyzing the dynamical properties of thermal receivers that the temperature is independent of the coordinate in the expression for the thermophysical coefficients $c[\theta(r,\tau)]$, $\gamma[\theta(r,\tau)]$, and $\lambda[\theta(r,\tau)]$. In this case, (1) becomes

$$\frac{\partial\theta(r,\ \tau)}{\partial\tau} = a(\tau)\left[\frac{\partial^2\theta(r,\ \tau)}{\partial r^2} + \frac{2p+1}{r}\ \frac{\partial\theta(r,\ \tau)}{\partial r}\right], \tag{2}$$

where

$$a(\tau) = \frac{\lambda(\tau)}{c(\tau)\,\gamma(\tau)}.$$

The equation obtained means that the initial nonlinear system has been reduced to a linear system with variable parameters. The dynamical properties of such a system are characterized completely by its transient, as follows from the theory of automatic regulation [9].

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Solving (2) by the Fourier method [6], we obtain an expression for the thermal-receiver transient along the medium temperature–thermal-receiver temperature channel after manipulations:

$$h_{\theta}(\beta,\ \tau) = 1 - \sum_{n=1}^{\infty} A_n(\mu_n) \frac{J_p(\beta\mu_n)}{\beta^p} \exp\left[-\frac{\mu_n^2}{R^2}\int_0^{\tau} a(\theta)\,d\tau\right]. \tag{3}$$

Here

$$\beta = r/R, \tag{4}$$

$$A_n(\mu_n) = \frac{2J_{p+1}(\mu_n)}{\mu_n\,[J_p^2(\mu_n) + J_{p+1}^2(\mu)] - 2pJ_p(\mu_n)\,J_{p+1}(\mu_n)}, \tag{5}$$

μ_n is a root of the characteristic equation

$$\frac{J_p(\mu)}{J_{p+1}(\mu)} = \frac{\mu}{\text{Bi}}\,\gamma_{\lambda}, \tag{6}$$

where

$$\gamma_{\lambda} = \frac{1}{\lambda_0(\theta_{\text{m}} - \theta_0)} \int_{\theta_0}^{\theta_{\text{m}}} \lambda(\theta)\,d\theta, \tag{7}$$

$$\text{Bi} = \frac{\alpha}{\lambda}\,R. \tag{8}$$

The equation [6] has been obtained as a result of linearizing the boundary condition, as is done by a competent application of the Fourier method [10, 11]. The error in determining the root of the characteristic equation because of the linearization will be estimated by the relationship

$$\frac{\Delta\mu_n}{\mu_n} = N_n(\mu_n)\left|\frac{\Delta\lambda(\theta)}{\lambda(\theta)}\right|, \tag{9}$$

where

$$N_n(\mu_n) = \frac{A_n(\mu_n)\,J_p(\mu_n)}{2}. \tag{10}$$

For the first root μ_1, the quantity $N_1(\mu_1)$ varies between 0.5 and zero as the Biot criterion goes from zero to infinity.

The time enters into the argument ξ in the expression obtained for the transient

$$\xi = \int_0^{\tau} a(\theta)\,d\tau \tag{11}$$

and performs the role of a "generalized" [12] time. It is easy to note that in the case of temperature-independence of the coefficient $a(\theta)$, known solutions of the linear problem of thermal conductivity [6] can be obtained from (3).

At this time, the inertial properties of thermal receivers are customarily estimated by the index of thermal inertia ε [13], which is a characteristic of the dynamical properties of the thermal receiver in the regular mode stage. In the case of thermal receivers with variable thermophysical parameters, the concepts of "regular mode" and of "index of thermal inertia" are meaningless, in which connection the need to introduce a criterion to estimate the inertial properties of thermal receivers originates.

It has been shown in [14] that the dynamical properties of a thermal receiver in both the preregular mode and the regular mode stages are characterized completely by a coordinate–time function $m(\beta, \tau)$ defined as

$$m(\beta,\ \tau) = -\frac{1}{\tau}\ln[1 - h_{\theta}(\beta,\ \tau)]. \tag{12}$$

The coordinate–time function is related to the index of thermal inertia by means of the relationship

$$\lim_{\tau\to\infty} m(\beta,\ \tau) = \frac{1}{\varepsilon}. \tag{13}$$

It is expedient for us to use the concept of the coordinate–time function to estimate the dynamical properties of thermal receivers with variable thermophysical parameters. In this case we obtain the following expression for $m(\beta, \tau)$:

$$m(\beta, \tau) = -\frac{1}{\tau} \ln \sum_{n=1}^{\infty} A_n(\mu_n) \frac{J_p(\beta\mu_n)}{\beta^p} \exp\left[-\frac{\mu_n^2}{R^2} \int_0^{\tau} a(\theta)\, d\tau\right]. \quad (14)$$

For large but finite times (14) becomes

$$m(\beta, \tau) = \frac{1}{\tau} \frac{\mu_1^2}{R^2} \int_0^{\tau} a(\theta)\, d\tau - \frac{1}{\tau} \ln A_1(\mu_1) \frac{J_p(\beta\mu_1)}{\beta^p}. \quad (15)$$

As $\tau \to \infty$

$$\lim_{\tau\to\infty} m(\beta, \tau) = \frac{\bar{a}_\theta \mu_1^2}{R^2}, \quad (16)$$

where

$$\bar{a}_\theta = \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^{\tau} a(\theta)\, d\tau. \quad (17)$$

The temperature dependence of the coefficient of thermal diffusivity can be represented to any accuracy required as the power series [15]

$$a(\theta) = a_0 \left[1 + \sum_{i=1}^{q} \alpha_i (\theta - \theta_0)\right], \quad (18)$$

where α_i are constant coefficients, and the time dependence of the temperature can be represented as an infinite series of exponential components,

$$\theta - \theta_0 = (\theta_m - \theta_0)\left[1 - \sum_{n=1}^{\infty} B_n \exp(-k_n\tau)\right], \quad (19)$$

where B_n and K_n are constant coefficients on which no constraints are imposed. Substituting (18) and (19) into (17), we obtain after manipulation

$$\lim_{\tau\to\infty} m(\beta, \tau) = \frac{a(\theta_m)\mu_1^2}{R^2}. \quad (20)$$

Let us introduce the notation

$$\lim_{\tau\to\infty} m(\beta, \tau) = \frac{1}{\bar{\varepsilon}} \quad (21)$$

and let us designate $\bar{\varepsilon}$ as the conditional index of thermal inertia.

The quantity $\bar{\varepsilon}$ characterizes the dynamical properties of thermal receivers as $\tau \to \infty$, i.e., in the quasistationary mode. This quantity is proposed as a criterion to estimate the inertia of thermal receivers. Evidently, $\bar{\varepsilon} = \varepsilon$ in the case of temperature independence of the thermophysical coefficients of a thermal receiver. An experimental determination of $\bar{\varepsilon}$ can be performed on the basis of (12) and (21), from which there follows that

$$\bar{\varepsilon} = -\lim_{\tau\to\infty} \frac{\tau}{\ln[1 - h_\theta(\beta, \tau)]}. \quad (22)$$

A pulse transient $V_\theta(\beta, \tau)$ over the medium temperature–thermal-receiver temperature channel can also be used to determine $\bar{\varepsilon}$ [16]. The computational formula is

$$\bar{\varepsilon} = -\lim_{\tau\to\infty} \frac{\tau}{\ln V_\theta(\beta, \tau)}. \quad (23)$$

It should be kept in mind in an experimental construction of the functions $h_\theta(\beta, \tau)$ and $V_\theta(\beta, \tau)$ that the magnitude of the step or pulse temperature effect on the thermal receiver should equal $\theta_m - \theta_0$.

NOTATION

τ is the time;
r is the coordinate;

c	is the specific heat;
γ	is the density;
λ and a	are the coefficients of thermal conductivity and diffusivity, respectively;
p	is the index of body shape;
R	is the cylinder or sphere radius or half the plate thickness;
Bi	is the Biot criterion;
J_p and J_{p+1}	are the Bessel functions of the first kind of order p and p + 1;
θ_0	is the initial temperature of the thermal receiver;
θ_m	is the temperature of the medium being checked;
θ	is the instantaneous value of the temperature;
Δ	is the absolute error.

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